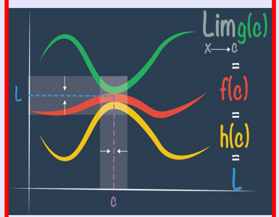
Math 261

Spring 2023

Lecture 13



Feb 19-8:47 AM

Class QZ 3:

Given $f(x) = x^2 - 8x$

1) find $f'(x)$ using the $\boxed{\text{def. of derivative}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) - x^2 + 8x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h - \cancel{x^2} + \cancel{8x}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + \overset{\to 0}{\cancel{h}} - 8)}{\cancel{h}} = \boxed{2x - 8}$$

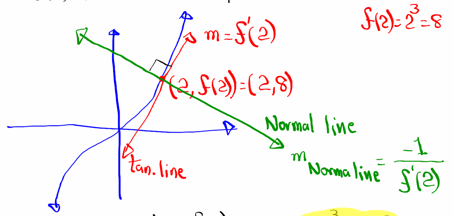
2) find x where $f'(x) = 0$.

$$2x - 8 = 0 \to \boxed{x = 4}$$

Feb 28-8:42 AM

Find equation of the normal line to the graph of $f(x) = x^3$ at the point where $x = 2$.

$f(2) = 2^3 = 8$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$$= \lim_{h \to 0} \frac{\cancel{h}\,((x+h)^2 + (x+h)x + x^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} \left[(x+h)^2 + (x+h)x + x^2\right] = x^2 + x^2 + x^2 = \boxed{3x^2}$$

$f'(2) = 3(2)^2 = 12 \rightarrow m_{\text{Normal line}} = \dfrac{-1}{12}$

Now eqn of the normal line

$\underbrace{y - y_1 = m(x - x_1)}_{\text{Point-Slope formula}} \Rightarrow y - 8 = \frac{-1}{12}(x - 2)$

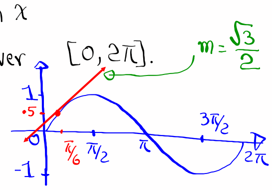
$y = \frac{-1}{12}x + \frac{1}{6} + 8$

$$\boxed{y = \frac{-1}{12}x + \frac{49}{6}}$$

Feb 28-9:12 AM

Given $f(x) = \sin x$

1) Graph $f(x)$ over $[0, 2\pi]$.



2) Find $f\left(\frac{\pi}{6}\right)$

$\frac{\pi}{6} = 30°$ $\quad f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

3) Find eqn of the tan. line to the graph of $f(x) = \sin x$ at $x = \frac{\pi}{6}$.

$m_{\text{tan. line}} = f'(\underset{\uparrow}{\ \ })$ $\quad = f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

tan. Point

From Yesterday's notes, we showed that

$f(x) = \sin x \rightarrow f'(x) = \cos x$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$y = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3}}{12} + \frac{1}{2} \Rightarrow \boxed{y = \frac{\sqrt{3}}{2}x + \frac{6 - \pi\sqrt{3}}{12}}$

Feb 28-9:23 AM

Find $\frac{d}{dx}[\cos x]$ using the def. of derivative.

$f(x) = \cos x \qquad f'(x)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{\cos x \cosh - \cos x} - \boxed{\sin x \sinh}}{h}$$

$$= \lim_{h \to 0} \frac{\cos x [\cosh - 1]}{h} - \lim_{h \to 0} \frac{\sin x \sinh}{h}$$

$$= \cos x \cdot \boxed{\lim_{h \to 0} \frac{\cosh - 1}{h}}^{\to 0} - \sin x \boxed{\lim_{h \to 0} \frac{\sinh}{h}}^{\to 1}$$

$$= \cancel{\cos x} \cdot 0 - \sin x \cdot 1 = \boxed{-\sin x}$$

$$\boxed{\frac{d}{dx}[\cos x] = -\sin x}, \quad \boxed{\frac{d}{dx}[\sin x] = \cos x}$$

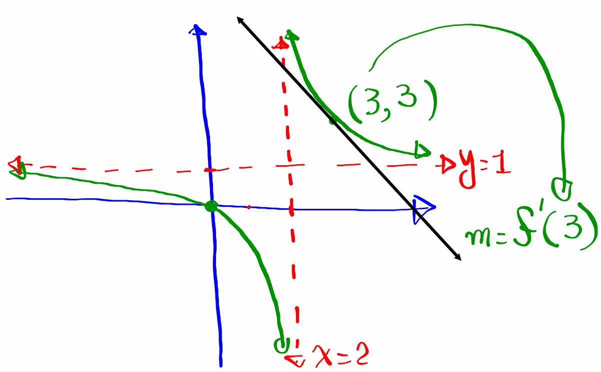
Feb 28-9:37 AM

Given $f(x) = \dfrac{x}{x-2}$ $\qquad f(3) = \dfrac{3}{3-2} = 3$

1) Domain $\quad x \neq 2 \quad (-\infty, 2) \cup (2, \infty)$

2) All intercepts. $\quad$ x-Int $(0,0)$ $\quad$ Y-Int $(0,0)$

3) All Asymptotes. $\quad$ Vertical Asymptote $x = 2$ $\quad$ Horizontal " $y = 1$



Feb 28-9:46 AM

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{3+h}{3+h-2} - 3}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3+h}{1+h} - 3}{h} = \lim_{h \to 0} \frac{3+h - 3(1+h)}{h(1+h)}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(1+\cancel{h}^{0})} = \boxed{-2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 3) \Rightarrow \boxed{y = -2x + 9}$$

Feb 28-9:53 AM